Sears and Zemansky's University Physics with Modern Physics Technology Update Hugh D. Young Roger A. Freedman Thirteenth Edition



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# PROBLEM SET: PERIODIC MOTION

Problems

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•, ••, •••: Problems of increasing difficulty. **CP**: Cumulative problems incorporating material from earlier chapters. **CALC**: Problems requiring calculus. **BIO**: Biosciences problems.

# **DISCUSSION QUESTIONS**

**Q1** An object is moving with SHM of amplitude *A* on the end of a spring. If the amplitude is doubled, what happens to the total distance the object travels in one period? What happens to the period? What happens to the maximum speed of the object? Discuss how these answers are related.

**Q2** Think of several examples in everyday life of motions that are, at least approximately, simple harmonic. In what respects does each differ from SHM?

**Q3** Does a tuning fork or similar tuning instrument undergo SHM? Why is this a crucial question for musicians?

**Q4** A box containing a pebble is attached to an ideal horizontal spring and is oscillating on a friction-free air table. When the box has reached its maximum distance from the equilibrium point, the pebble is suddenly lifted out vertically without disturbing the box. Will the following characteristics of the motion increase, decrease, or remain the same in the subsequent motion of the box? Justify each answer. (a) frequency; (b) period; (c) amplitude; (d) the maxi-

**Q5** If a uniform spring is cut in half, what is the force constant of each half? Justify your answer. How would the frequency of SHM using a half-spring differ from the frequency using the same mass and the entire spring?

**Q6** The analysis of SHM in this chapter ignored the mass of the spring. How does the spring's mass change the characteristics of the motion?

**Q7** Two identical gliders on an air track are connected by an ideal spring. Could such a system undergo SHM? Explain. How would the period compare with that of a single glider attached to a spring whose other end is rigidly attached to a stationary object? Explain. **Q8** You are captured by Martians, taken into their ship, and put to sleep. You awake some time later and find yourself locked in a small room with no windows. All the Martians have left you with is your digital watch, your school ring, and your long silver-chain necklace. Explain how you can determine whether you are still on earth or have been transported to Mars.

**Q9** The system shown in Fig. 17 is mounted in an elevator. What happens to the period of the motion (does it increase, decrease, or remain the same) if the elevator (a) accelerates upward at  $5.0 \text{ m/s}^2$ ; (b) moves upward at a steady 5.0 m/s; (c) accelerates downward at  $5.0 \text{ m/s}^2$ ? Justify your answers.





Tines with small mass m: <sup>/</sup> high frequency f = 4096 Hz

**Q10** If a pendulum has a period of 2.5 s on earth, what would be its period in a space station orbiting the earth? If a mass hung from a vertical spring has a period of 5.0 s on earth, what would its period be in the space station? Justify each of your answers.

**Q11** A simple pendulum is mounted in an elevator. What happens to the period of the pendulum (does it increase, decrease, or remain the same) if the elevator (a) accelerates upward at  $5.0 \text{ m/s}^2$ ; (b) moves upward at a steady 5.0 m/s; (c) accelerates downward at  $5.0 \text{ m/s}^2$ ; (d) accelerates downward at  $9.8 \text{ m/s}^2$ ? Justify your answers.

**Q12** What should you do to the length of the string of a simple pendulum to (a) double its frequency; (b) double its period; (c) double its angular frequency?

**Q13** If a pendulum clock is taken to a mountaintop, does it gain or lose time, assuming it is correct at a lower elevation? Explain your answer.

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**Q14** When the amplitude of a simple pendulum increases, should its period increase or decrease? Give a qualitative argument. Is your argument also valid for a physical pendulum?

**Q15** Why do short dogs (like Chihuahuas) walk with quicker strides than do tall dogs (like Great Danes)?

**Q16** At what point in the motion of a simple pendulum is the string tension greatest? Least? In each case give the reasoning behind your answer.

**Q17** Could a standard of time be based on the period of a certain standard pendulum? What advantages and disadvantages would such a standard have compared to the actual present-day standard.

**Q18** For a simple pendulum, clearly distinguish between  $\omega$  (the angular velocity) and  $\omega$  (the angular frequency). Which is constant and which is variable?

**Q19** A glider is attached to a fixed ideal spring and oscillates on a horizontal, friction-free air track. A coin is atop the glider and oscillating with it. At what points in the motion is the friction force on the coin greatest? At what points is it least? Justify your answers.

**Q20** In designing structures in an earthquake-prone region, how should the natural frequencies of oscillation of a structure relate to typical earthquake frequencies? Why? Should the structure have a large or small amount of damping?

# **EXERCISES**

#### Section 1 Describing Oscillation

**1** • **BIO** (a) **Music.** When a person sings, his or her vocal cords vibrate in a repetitive pattern that has the same frequency as the note that is sung. If someone sings the note B flat, which has a frequency of 466 Hz, how much time does it take the person's vocal cords to vibrate through one complete cycle, and what is the angular frequency of the cords? (b) Hearing. When sound waves strike the eardrum, this membrane vibrates with the same frequency as the sound. The highest pitch that typical humans can hear has a period of 50.0  $\mu$ s. What are the frequency and angular frequency of the vibrating eardrum for this sound? (c) Vision. When light having vibrations with angular frequency ranging from  $2.7 \times 10^{15}$  rad/s to  $4.7 \times 10^{15}$  rad/s strikes the retina of the eye, it stimulates the receptor cells there and is perceived as visible light. What are the limits of the period and frequency of this light? (d) Ultrasound. High-frequency sound waves (ultrasound) are used to probe the interior of the body, much as x rays do. To detect small objects such as tumors, a frequency of around 5.0 MHz is used. What are the period and angular frequency of the molecular vibrations caused by this pulse of sound?

**2** • If an object on a horizontal, frictionless surface is attached to a spring, displaced, and then released, it will oscillate. If it is displaced 0.120 m from its equilibrium position and released with zero initial speed, then after 0.800 s its displacement is found to be 0.120 m on the opposite side, and it has passed the equilibrium position once during this interval. Find (a) the amplitude; (b) the period; (c) the frequency.

**3** • The tip of a tuning fork goes through 440 complete vibrations in 0.500 s. Find the angular frequency and the period of the motion.

**4** • The displacement of an oscillating object as a function of time is shown in Fig. E4. What are (a) the frequency; (b) the amplitude; (c) the period; (d) the angular frequency of this motion?

Figure **E4** 



**5** •• A machine part is undergoing SHM with a frequency of 5.00 Hz and amplitude 1.80 cm. How long does it take the part to go from x = 0 to x = -1.80 cm?

### **Section 2 Simple Harmonic Motion**

**\mathbf{6}** •• In a physics lab, you attach a 0.200-kg air-track glider to the end of an ideal spring of negligible mass and start it oscillating. The elapsed time from when the glider first moves through the equilibrium point to the second time it moves through that point is 2.60 s. Find the spring's force constant.

**7** • When a body of unknown mass is attached to an ideal spring with force constant 120 N/m, it is found to vibrate with a frequency of 6.00 Hz. Find (a) the period of the motion; (b) the angular frequency; (c) the mass of the body.

**8** • When a 0.750-kg mass oscillates on an ideal spring, the frequency is 1.33 Hz. What will the frequency be if 0.220 kg are (a) added to the original mass and (b) subtracted from the original mass? Try to solve this problem *without* finding the force constant of the spring.

**9** •• An object is undergoing SHM with period 0.900 s and amplitude 0.320 m. At t = 0 the object is at x = 0.320 m and is instantaneously at rest. Calculate the time it takes the object to go (a) from x = 0.320 m to x = 0.160 m and (b) from x = 0.160 m to x = 0.

**10** • A small block is attached to an ideal spring and is moving in SHM on a horizontal, frictionless surface. When the block is at x = 0.280 m, the acceleration of the block is -5.30 m/s<sup>2</sup>. What is the frequency of the motion?

**11** • A 2.00-kg, frictionless block is attached to an ideal spring with force constant 300 N/m. At t = 0 the spring is neither stretched nor compressed and the block is moving in the negative direction at 12.0 m/s. Find (a) the amplitude and (b) the phase angle. (c) Write an equation for the position as a function of time.

**12** •• Repeat Exercise 11, but assume that at t = 0 the block has velocity -4.00 m/s and displacement +0.200 m.

**13** • The point of the needle of a sewing machine moves in SHM along the *x*-axis with a frequency of 2.5 Hz. At t = 0 its position and velocity components are +1.1 cm and -15 cm/s, respectively. (a) Find the acceleration component of the needle at t = 0. (b) Write equations giving the position, velocity, and acceleration components of the point as a function of time.

**14** •• A small block is attached to an ideal spring and is moving in SHM on a horizontal, frictionless surface. When the amplitude of the motion is 0.090 m, it takes the block 2.70 s to travel from x = 0.090 m to x = -0.090 m. If the amplitude is doubled, to 0.180 m, how long does it take the block to travel (a) from x = 0.180 m to x = -0.180 m and (b) from x = 0.090 m to x = -0.090 m?

**15** • **BIO** Weighing Astronauts. This procedure has actually been used to "weigh" astronauts in space. A 42.5-kg chair is

attached to a spring and allowed to oscillate. When it is empty, the chair takes 1.30 s to make one complete vibration. But with an astronaut sitting in it, with her feet off the floor, the chair takes 2.54 s for one cycle. What is the mass of the astronaut?

**16** • A 0.400-kg object undergoing SHM has  $a_x = -2.70 \text{ m/s}^2$  when x = 0.300 m. What is the time for one oscillation?

**17** • On a frictionless, horizontal air track, a glider oscillates at the end of an ideal spring of force constant 2.50 N/cm. The graph in Fig. E17 shows the acceleration of the glider as a function of time. Find (a) the mass of the glider; (b) the maximum displacement of the





glider from the equilibrium point; (c) the maximum force the spring exerts on the glider.

**18** • A 0.500-kg mass on a spring has velocity as a function of time given by  $v_x(t) = -(3.60 \text{ cm/s}) \sin[(4.71 \text{ s}^{-1})t - \pi/2]$ . What are (a) the period; (b) the amplitude; (c) the maximum acceleration of the mass; (d) the force constant of the spring?

**19** • A 1.50-kg mass on a spring has displacement as a function of time given by the equation

$$x(t) = (7.40 \text{ cm}) \cos[(4.16 \text{ s}^{-1})t - 2.42]$$

Find (a) the time for one complete vibration; (b) the force constant of the spring; (c) the maximum speed of the mass; (d) the maximum force on the mass; (e) the position, speed, and acceleration of the mass at t = 1.00 s; (f) the force on the mass at that time.

**20** • **BIO** Weighing a Virus. In February 2004, scientists at Purdue University used a highly sensitive technique to measure the mass of a vaccinia virus (the kind used in smallpox vaccine). The procedure involved measuring the frequency of oscillation of a tiny sliver of silicon (just 30 nm long) with a laser, first without the virus and then after the virus had attached itself to the silicon. The difference in mass caused a change in the frequency. We can model such a process as a mass on a spring. (a) Show that the ratio of the frequency with the virus attached ( $f_{S+V}$ ) to the frequency without

the virus  $(f_{\rm S})$  is given by the formula  $\frac{f_{\rm S+V}}{f_{\rm S}} = \frac{1}{\sqrt{1 + (m_{\rm V}/m_{\rm S})}}$ 

where  $m_V$  is the mass of the virus and  $m_S$  is the mass of the silicon sliver. Notice that it is *not* necessary to know or measure the force constant of the spring. (b) In some data, the silicon sliver has a mass of  $2.10 \times 10^{-16}$  g and a frequency of  $2.00 \times 10^{15}$  Hz without the virus and  $2.87 \times 10^{14}$  Hz with the virus. What is the mass of the virus, in grams and in femtograms?

**21** •• **CALC** Jerk. A guitar string vibrates at a frequency of 440 Hz. A point at its center moves in SHM with an amplitude of 3.0 mm and a phase angle of zero. (a) Write an equation for the position of the center of the string as a function of time. (b) What are the maximum values of the magnitudes of the velocity and acceleration of the center of the string? (c) The derivative of the acceleration with respect to time is a quantity called the *jerk*. Write an equation for the jerk of the center of the string as a function of time, and find the maximum value of the magnitude of the jerk.

#### Section 3 Energy in Simple Harmonic Motion

**22** •• For the oscillating object in Fig. E4, what are (a) its maximum speed and (b) its maximum acceleration?

**23** • A small block is attached to an ideal spring and is moving in SHM on a horizontal, frictionless surface. The amplitude of the

motion is 0.120 m. The maximum speed of the block is 3.90 m/s. What is the maximum magnitude of the acceleration of the block?

**24** • A small block is attached to an ideal spring and is moving in SHM on a horizontal, frictionless surface. The amplitude of the motion is 0.250 m and the period is 3.20 s. What are the speed and acceleration of the block when x = 0.160 m?

**25** •• A tuning fork labeled 392 Hz has the tip of each of its two prongs vibrating with an amplitude of 0.600 mm. (a) What is the maximum speed of the tip of a prong? (b) A housefly (*Musca domestica*) with mass 0.0270 g is holding onto the tip of one of the prongs. As the prong vibrates, what is the fly's maximum kinetic energy? Assume that the fly's mass has a negligible effect on the frequency of oscillation.

**26** •• A harmonic oscillator has angular frequency  $\omega$  and amplitude A. (a) What are the magnitudes of the displacement and velocity when the elastic potential energy is equal to the kinetic energy? (Assume that U = 0 at equilibrium.) (b) How often does this occur in each cycle? What is the time between occurrences? (c) At an instant when the displacement is equal to A/2, what fraction of the total energy of the system is kinetic and what fraction is potential?

**27** • A 0.500-kg glider, attached to the end of an ideal spring with force constant k = 450 N/m, undergoes SHM with an amplitude of 0.040 m. Compute (a) the maximum speed of the glider; (b) the speed of the glider when it is at x = -0.015 m; (c) the magnitude of the maximum acceleration of the glider; (d) the acceleration of the glider at x = -0.015 m; (e) the total mechanical energy of the glider at any point in its motion.

**28** •• A cheerleader waves her pom-pom in SHM with an amplitude of 18.0 cm and a frequency of 0.850 Hz. Find (a) the maximum magnitude of the acceleration and of the velocity; (b) the acceleration and speed when the pom-pom's coordinate is x = +9.0 cm; (c) the time required to move from the equilibrium position directly to a point 12.0 cm away. (d) Which of the quantities asked for in parts (a), (b), and (c) can be found using the energy approach, and which cannot? Explain.

**29** • **CP** For the situation described in part (a) of Example 5, what should be the value of the putty mass m so that the amplitude after the collision is one-half the original amplitude? For this value of m, what fraction of the original mechanical energy is converted into heat?

**30** • A 0.150-kg toy is undergoing SHM on the end of a horizontal spring with force constant k = 300 N/m. When the object is 0.0120 m from its equilibrium position, it is observed to have a speed of 0.300 m/s. What are (a) the total energy of the object at any point of its motion; (b) the amplitude of the motion; (c) the maximum speed attained by the object during its motion?

**31** •• You are watching an object that is moving in SHM. When the object is displaced 0.600 m to the right of its equilibrium position, it has a velocity of 2.20 m/s to the right and an acceleration of 8.40 m/s<sup>2</sup> to the left. How much farther from this point will the object move before it stops momentarily and then starts to move back to the left?

**32** •• On a horizontal, frictionless table, an open-topped 5.20-kg box is attached to an ideal horizontal spring having force constant 375 N/m. Inside the box is a 3.44-kg stone. The system is oscillating with an amplitude of 7.50 cm. When the box has reached its maximum speed, the stone is suddenly plucked vertically out of the box without touching the box. Find (a) the period and (b) the amplitude of the resulting motion of the box. (c) Without doing

any calculations, is the new period greater or smaller than the original period? How do you know?

**33** •• A mass is oscillating with amplitude A at the end of a spring. How far (in terms of A) is this mass from the equilibrium position of the spring when the elastic potential energy equals the kinetic energy?

**34** •• A mass m is attached to a spring of force constant 75 N/m and allowed to oscillate. Figure E34 shows a graph of its velocity  $v_r$  as a function of time t. Find (a) the period, (b) the frequency, and (c) the angular fre-



quency of this motion. (d) What is the amplitude (in cm), and at what times does the mass reach this position? (e) Find the maximum acceleration of the mass and the times at which it occurs. (f) What is the mass *m*?

**35** • Inside a NASA test vehicle, a 3.50-kg ball is pulled along by a horizontal ideal spring fixed to a friction-free table. The force constant of the spring is 225 N/m. The vehicle has a steady acceleration of 5.00 m/s<sup>2</sup>, and the ball is not oscillating. Suddenly, when the vehicle's speed has reached 45.0 m/s, its engines turn off, thus eliminating its acceleration but not its velocity. Find (a) the amplitude and (b) the frequency of the resulting oscillations of the ball. (c) What will be the ball's maximum speed relative to the vehicle?

#### Section 4 Applications of Simple Harmonic Motion

**36** • A proud deep-sea fisherman hangs a 65.0-kg fish from an ideal spring having negligible mass. The fish stretches the spring 0.120 m. (a) Find the force constant of the spring. The fish is now pulled down 5.00 cm and released. (b) What is the period of oscillation of the fish? (c) What is the maximum speed it will reach?

**37** • A 175-g glider on a horizontal, frictionless air track is attached to a fixed ideal spring with force constant 155 N/m. At the instant you make measurements on the glider, it is moving at 0.815 m/s and is 3.00 cm from its equilibrium point. Use *energy* conservation to find (a) the amplitude of the motion and (b) the maximum speed of the glider. (c) What is the angular frequency of the oscillations?

**38** • A thrill-seeking cat with mass 4.00 kg is attached by a harness to an ideal spring of negligible mass and oscillates vertically in SHM. The amplitude is 0.050 m, and at the highest point of the motion the spring has its natural unstretched length. Calculate the elastic potential energy of the spring (take it to be zero for the unstretched spring), the kinetic energy of the cat, the gravitational potential energy of the system relative to the lowest point of the motion, and the sum of these three energies when the cat is (a) at its highest point; (b) at its lowest point; (c) at its equilibrium position.

**39** • A 1.50-kg ball and a 2.00-kg ball are glued together with the lighter one below the heavier one. The upper ball is attached to a vertical ideal spring of force constant 165 N/m, and the system is vibrating vertically with amplitude 15.0 cm. The glue connecting the balls is old and weak, and it suddenly comes loose when the balls are at the lowest position in their motion. (a) Why is the glue more likely to fail at the *lowest* point than at any other point in the motion? (b) Find the amplitude and frequency of the vibrations after the lower ball has come loose.

**40** •• A uniform, solid metal disk of mass 6.50 kg and diameter 24.0 cm hangs in a horizontal plane, supported at its center by a vertical metal wire. You find that it requires a horizontal force of 4.23 N tangent to the rim of the disk to turn it by 3.34°, thus twisting the wire. You now remove this force and release the disk from rest. (a) What is the torsion constant for the metal wire? (b) What are the frequency and period of the torsional oscillations of the disk? (c) Write the equation of motion for  $\theta(t)$  for the disk.

**41** •• A certain alarm clock ticks four times each second, with each tick representing half a period. The balance wheel consists of a thin rim with radius 0.55 cm, connected to the balance staff by thin spokes of negligible mass. The total mass of the balance wheel is 0.90 g. (a) What is the moment of inertia of the balance wheel about its shaft? (b) What is the torsion constant of the coil spring?

Figure E42

**42** • A thin metal disk with mass  $2.00 \times 10^{-3}$  kg and radius 2.20 cm is attached at its center to a long fiber (Fig. E42). The disk, when twisted and released, oscillates with a period of 1.00 s. Find the torsion constant of the fiber.



plicated machine part about an axis through its center of mass. You suspend it from a wire along this axis. The wire has a torsion constant of 0.450 N · m/rad. You twist the part a small amount about this axis and let it go, timing 125 oscillations in 265 s. What is the moment of inertia you want to find?

**44** •• **CALC** The balance wheel of a watch vibrates with an angular amplitude  $\Theta$ , angular frequency  $\omega$ , and phase angle  $\phi = 0$ . (a) Find expressions for the angular velocity  $d\theta/dt$  and angular acceleration  $d^2\theta/dt^2$  as functions of time. (b) Find the balance wheel's angular velocity and angular acceleration when its angular displacement is  $\Theta$ , and when its angular displacement is  $\Theta/2$  and  $\theta$  is decreasing. (*Hint:* Sketch a graph of  $\theta$  versus *t*.)

#### Section 5 The Simple Pendulum

45 •• You pull a simple pendulum 0.240 m long to the side through an angle of 3.50° and release it. (a) How much time does it take the pendulum bob to reach its highest speed? (b) How much time does it take if the pendulum is released at an angle of 1.75° instead of 3.50°?

**46** • An 85.0-kg mountain climber plans to swing down, starting from rest, from a ledge using a light rope 6.50 m long. He holds one end of the rope, and the other end is tied higher up on a rock face. Since the ledge is not very far from the rock face, the rope makes a small angle with the vertical. At the lowest point of his swing, he plans to let go and drop a short distance to the ground. (a) How long after he begins his swing will the climber first reach his lowest point? (b) If he missed the first chance to drop off, how long after first beginning his swing will the climber reach his lowest point for the second time?

47 • A building in San Francisco has light fixtures consisting of small 2.35-kg bulbs with shades hanging from the ceiling at the end of light, thin cords 1.50 m long. If a minor earthquake occurs, how many swings per second will these fixtures make?

48 • A Pendulum on Mars. A certain simple pendulum has a period on the earth of 1.60 s. What is its period on the surface of Mars, where  $g = 3.71 \text{ m/s}^2$ ?

**49** • After landing on an unfamiliar planet, a space explorer constructs a simple pendulum of length 50.0 cm. She finds that the pendulum makes 100 complete swings in 136 s. What is the value of g on this planet?