University Physics

with Modern Physics

Fifteenth Edition in SI Units

Hugh D. Young • Roger A. Freedman

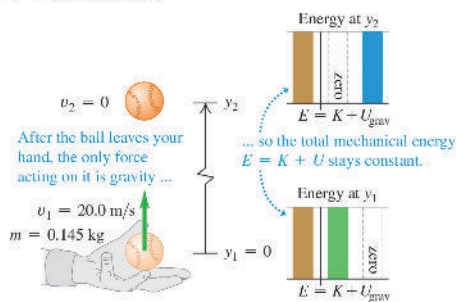# Guided practice features to help...

EXAMPLE 7.1 Height of a baseball from energy conservation
WITH √ARIATION PROBLEMS

You throw a 0.145 kg baseball straight up, giving it an initial velocity of magnitude 20.0 m/s. Find how high it goes, ignoring air resistance.

IDENTIFY and SET UP After the ball leaves your hand, only gravity does work on it. Hence total mechanical energy is conserved, and we can use Eq. (7.4). We take point 1 to be where the ball leaves your hand and point 2 to be where it reaches its maximum height. As in Fig. 7.2, we take the positive y-direction to be upward. The ball's speed at point 1 is $v_1 = 20.0$ m/s; at its maximum height it is instantaneously at rest, so $v_2 = 0$. We take the origin at point 1, so $y_1 = 0$ (**Fig. 7.4**). Our target variable, the distance the ball moves vertically between the two points, is the displacement $y_2 - y_1 = y_2 - 0 = y_2$.

Figure 7.4 After a baseball leaves your hand, total mechanical energy $E = K + U$ is conserved.

After the ball leaves your hand, the only force acting on it is gravity ...

... so the total mechanical energy $E = K + U$ stays constant.

$v_2 = 0$
$v_1 = 20.0$ m/s
$m = 0.145$ kg

EXECUTE We have $y_1 = 0$, $U_{grav,1} = mgy_1 = 0$, and $K_2 = \frac{1}{2}mv_2^2 = 0$. Then Eq. (7.4), $K_1 + U_{grav,1} = K_2 + U_{grav,2}$, becomes

$$K_1 = U_{grav,2}$$

As the energy bar graphs in Fig. 7.4 show, this equation says that the kinetic energy of the ball at point 1 is completely converted to gravitational potential energy at point 2. We substitute $K_1 = \frac{1}{2}mv_1^2$ and $U_{grav,2} = mgy_2$ and solve for y_2:

$$\tfrac{1}{2}mv_1^2 = mgy_2$$

$$y_2 = \frac{v_1^2}{2g} = \frac{(20.0\ \text{m/s})^2}{2(9.80\ \text{m/s}^2)} = 20.4\ \text{m}$$

EVALUATE As a check, use the given value of v_1 and our result for y_2 to calculate the kinetic energy at point 1 and the gravitational potential energy at point 2. You should find that these are equal: $K_1 = \frac{1}{2}mv_1^2 = 29.0$ J and $U_{grav,2} = mgy_2 = 29.0$ J. Note that we could have found the result $y_2 = v_1^2/2g$ by using Eq. (2.13) in the form $v_{2y}^2 = v_{1y}^2 - 2g(y_2 - y_1)$.

What if we put the origin somewhere else—for example, 5.0 m below point 1, so that $y_1 = 5.0$ m? Then the total mechanical energy at point 1 is part kinetic and part potential; at point 2 it's still purely potential because $v_2 = 0$. You'll find that this choice of origin yields $y_2 = 25.4$ m, but again $y_2 - y_1 = 20.4$ m. In problems like this, you are free to choose the height at which $U_{grav} = 0$. The physics doesn't depend on your choice.

▶ **KEY CONCEPT** Total mechanical energy (the sum of kinetic energy and gravitational potential energy) is conserved when only the force of gravity does work.

GUIDED PRACTICE

KEY EXAMPLE √ARIATION PROBLEMS

Be sure to review **EXAMPLES 7.1 and 7.2** (Section 7.1) before attempting these problems.

VP7.2.1 You throw a cricket ball (mass 0.156 kg) vertically upward. It leaves your hand moving at 12.0 m/s. Air resistance can be neglected. At what height above your hand does the ball have (a) half as much upward velocity, (b) half as much kinetic energy as when it left your hand?
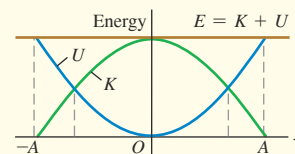
VP7.2.2 You toss a rock of mass m vertically upward. Air resistance can be neglected. The rock reaches a maximum height h above your hand. What is the speed of the rock when it is at height (a) $h/4$ and (b) $3h/4$?

VP7.2.3 You throw a tennis ball (mass 0.0570 kg) vertically upward. It leaves your hand moving at 15.0 m/s. Air resistance cannot be neglected, and the ball reaches a maximum height of 8.00 m. (a) By how much does the total mechanical energy decrease from when the ball leaves your hand to when it reaches its maximum height? (b) What is the magnitude of the average force of air resistance?

VP7.2.4 You catch a volleyball (mass 0.270 kg) that is moving downward at 7.50 m/s. In stopping the ball, your hands and the volleyball descend together a distance of 0.150 m. (a) How much work do your hands do on the volleyball in the process of stopping it? (b) What is the magnitude of the force (assumed constant) that your hands exert on the volleyball?
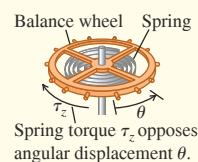
Energy in simple harmonic motion: Energy is conserved in SHM. The total energy can be expressed in terms of the force constant k and amplitude A. (See Examples 14.4 and 14.5.)

$$E = \tfrac{1}{2}mv_x^2 + \tfrac{1}{2}kx^2$$
$$= \tfrac{1}{2}kA^2 = \text{constant}$$

(14.21)

Angular simple harmonic motion: In angular SHM, the frequency and angular frequency are related to the moment of inertia I and the torsion constant κ.

$$\omega = \sqrt{\frac{\kappa}{I}} \quad \text{and}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{\kappa}{I}}$$
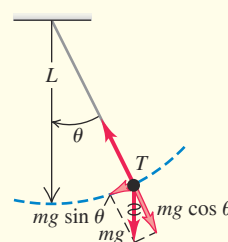
(14.24)

Simple pendulum: A simple pendulum consists of a point mass m at the end of a massless string of length L. Its motion is approximately simple harmonic for sufficiently small amplitude; the angular frequency, frequency, and period then depend on only g and L, not on the mass or amplitude. (See Example 14.8.)

$$\omega = \sqrt{\frac{g}{L}}$$ (14.32)

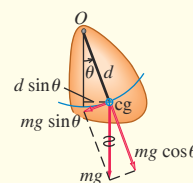$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$ (14.33)

$$T = \frac{2\pi}{\omega} = \frac{1}{f} = 2\pi\sqrt{\frac{L}{g}}$$ (14.34)

Physical pendulum: A physical pendulum is any object suspended from an axis of rotation. The angular frequency and period for small-amplitude oscillations are independent of amplitude but depend on the mass m, distance d from the axis of rotation to the center of gravity, and moment of inertia I about the axis. (See Examples 14.9 and 14.10.)
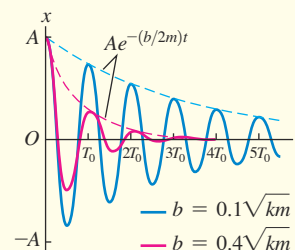
$$\omega = \sqrt{\frac{mgd}{I}}$$ (14.38)

$$T = 2\pi\sqrt{\frac{I}{mgd}}$$ (14.39)

Damped oscillations: When a force $F_x = -bv_x$ is added to a simple harmonic oscillator, the motion is called a damped oscillation. If $b < 2\sqrt{km}$ (called underdamping), the system oscillates with a decaying amplitude and an angular frequency ω' that is lower than it would be without damping. If $b = 2\sqrt{km}$ (called critical damping) or $b > 2\sqrt{km}$ (called overdamping), when the system is displaced it returns to equilibrium without oscillating.

$$x = Ae^{-(b/2m)t}\cos(\omega' t + \phi)$$ (14.42)

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$ (14.43)

Forced oscillations and resonance: When a sinusoidally varying driving force is added to a damped harmonic oscillator, the resulting motion is called a forced oscillation or driven oscillation. The amplitude is a function of the driving frequency ω_d and reaches a peak at a driving frequency close to the natural frequency of the system. This behavior is called resonance.

$$A = \frac{F_{max}}{\sqrt{(k - m\omega_d^2)^2 + b^2\omega_d^2}}$$

(14.46)

GUIDED PRACTICE

For assigned homework and other learning materials, go to Mastering Physics.

KEY EXAMPLE √ARIATION PROBLEMS

Be sure to review EXAMPLES 14.2 and 14.3 (Section 14.2) before attempting these problems.

VP14.3.1 A glider of mass 0.400 kg is placed on a frictionless, horizontal air track. One end of a horizontal spring is attached to the glider, and the other end is attached to the end of the track. When released, the glider oscillates in SHM with frequency 4.15 Hz. (a) Find the period and angular frequency of the motion. (b) Find the force constant k of the spring. (c) Find the magnitude of the force that the spring exerts on the glider when the spring is stretched by 0.0200 m.

VP14.3.2 A hockey puck attached to a horizontal spring oscillates on a frictionless, horizontal surface. The spring has force constant 4.50 N/m and the oscillation period is 1.20 s. (a) What is the mass of the puck? (b) During an oscillation, the acceleration of the puck has maximum magnitude 1.20 m/s². What is the amplitude of the oscillation?

VP14.3.3 The piston of a petrol engine oscillates in SHM with frequency 50.0 Hz. At one point in the cycle the piston is 0.0300 m from equilibrium and moving at 12.4 m/s. (a) What is the amplitude of the motion? (b) What is the maximum speed the piston attains during its oscillation?

VP14.3.4 A cat is sleeping on a platform that oscillates from side to side in SHM. The combined mass of the cat and platform is 5.00 kg, and the force constant of the horizontal spring attached to the platform that makes it oscillate is 185 N/m. Ignore friction. (a) What is the frequency of the oscillation? (b) The cat will wake up if the acceleration of the platform is greater than 1.52 m/s². What is the maximum amplitude of oscillation that will allow the cat to stay asleep?

Be sure to review EXAMPLE 14.4 (Section 14.3) before attempting these problems.

VP14.4.1 A hockey puck oscillates on a frictionless, horizontal track while attached to a horizontal spring. The puck has mass 0.150 kg and the spring has force constant 8.00 N/m. The maximum speed of the puck during its oscillation is 0.350 m/s. (a) What is the amplitude of the oscillation? (b) What is the total mechanical energy of the oscillation?

(c) What are the potential energy and the kinetic energy of the puck when the displacement of the glider is 0.0300 m?

VP14.4.2 A block of mass 0.300 kg attached to a horizontal spring oscillates on a frictionless surface. The oscillation has amplitude 0.0440 m, and total mechanical energy $E = 6.00 \times 10^{-2}$ J. Find (a) the force constant of the spring and (b) the block's speed when the potential energy equals exactly $E/2$.

VP14.4.3 A glider attached to a horizontal spring oscillates on a horizontal air track. The total mechanical energy of the oscillation is 4.00×10^{-3} J, the amplitude of the oscillation is 0.0300 m, and the maximum speed of the glider is 0.125 m/s. (a) What are the force constant of the spring and the mass of the glider? (b) What is the maximum acceleration of the glider? (c) What is the magnitude of the glider's acceleration when the potential energy equals 3.00×10^{-3} J?

VP14.4.4 An object is undergoing SHM with amplitude A. For what values of the displacement is the kinetic energy equal to (a) $\frac{1}{3}$ of the total mechanical energy; (b) $\frac{4}{5}$ of the total mechanical energy?

Be sure to review EXAMPLES 14.8 and 14.9 (Sections 14.5 and 14.6) before attempting these problems.

VP14.9.1 On an alien planet, a simple pendulum of length 0.500 m has oscillation frequency 0.609 Hz. Find (a) the period of the pendulum and (b) the acceleration due to gravity on this planet's surface.

VP14.9.2 What must be the length of a simple pendulum if its oscillation frequency is to be equal to that of an air-track glider of mass 0.350 kg attached to a spring of force constant 8.75 N/m?

VP14.9.3 At a bicycle repair shop, a bicycle tire of mass M and radius R is suspended from a peg on the wall. The moment of inertia of the tire around the peg is $2MR^2$. If the tire is displaced from equilibrium and starts swinging back and forth, what will be its frequency of oscillation?

VP14.9.4 A rod has length 0.900 m and mass 0.600 kg and is pivoted at one end. The rod is *not* uniform; the center of mass of the rod is not at its center but is 0.500 m from the pivot. The period of the rod's motion as a pendulum is 1.59 s. What is the moment of inertia of the rod around the pivot?
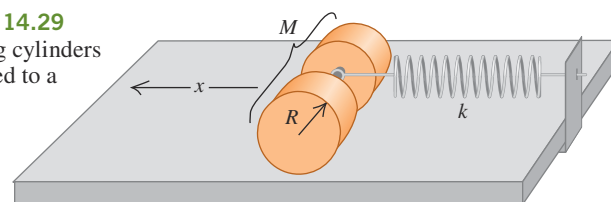
BRIDGING PROBLEM Oscillating and Rolling

Two uniform, solid cylinders of radius R and total mass M are connected along their common axis by a short, light rod and rest on a horizontal tabletop (**Fig. 14.29**). A frictionless ring at the rod's center is attached to a spring of force constant k; the spring's other end is fixed. The cylinders are pulled to the left a distance x, stretching the spring, then released from rest. Due to friction between the tabletop and the cylinders, the cylinders roll without slipping as they oscillate. Show that the motion of the center of mass of the cylinders is simple harmonic, and find its period.

Figure **14.29**
Rolling cylinders attached to a spring.

SOLUTION GUIDE

IDENTIFY and SET UP

1. What condition must be satisfied for the motion of the center of mass of the cylinders to be simple harmonic?

2. Which equations should you use to describe (a) the translational and rotational motions of the cylinders; (b) Which equation should you use to describe the condition that the cylinders roll without slipping? (*Hint:* See Section 10.3.)

3. Sketch the situation and choose a coordinate system. List the unknown quantities and decide which is the target variable.

EXECUTE

4. Draw a free-body diagram for the cylinders when they are displaced a distance x from equilibrium.

5. Solve the equations to find an expression for the acceleration of the center of mass of the cylinders. What does this expression tell you?

6. Use your result from step 5 to find the period of oscillation of the center of mass of the cylinders.

EVALUATE

7. What would be the period of oscillation if there were no friction and the cylinders didn't roll? Is this period larger or smaller than your result from step 6? Is this reasonable?

PROBLEMS

•, ••, •••: Difficulty levels. **CP**: Cumulative problems incorporating material from earlier chapters. **CALC**: Problems requiring calculus.
DATA: Problems involving real data, scientific evidence, experimental design, and/or statistical reasoning. **BIO**: Biosciences problems.

DISCUSSION QUESTIONS

Q14.1 An object is moving with SHM of amplitude A on the end of a spring. If the amplitude is doubled, what happens to the total distance the object travels in one period? What happens to the period? What happens to the maximum speed of the object? Discuss how these answers are related.

Q14.2 Think of several examples in everyday life of motions that are, at least approximately, simple harmonic. In what respects does each differ from SHM?

Q14.3 Does a tuning fork or similar tuning instrument undergo SHM? Why is this a crucial question for musicians?

Q14.4 A box containing a pebble is attached to an ideal horizontal spring and is oscillating on a friction-free air table. When the box has reached its maximum distance from the equilibrium point, the pebble is suddenly lifted out vertically without disturbing the box. Will the following characteristics of the motion increase, decrease, or remain the same in the subsequent motion of the box? Justify each answer. (a) Frequency; (b) period; (c) amplitude; (d) the maximum kinetic energy of the box; (e) the maximum speed of the box.

Q14.5 If a uniform spring is cut in half, what is the force constant of each half? Justify your answer. How would the frequency of SHM using a half-spring differ from the frequency using the same mass and the entire spring?

Q14.6 A glider is attached to a fixed ideal spring and oscillates on a horizontal, friction-free air track. A coin rests atop the glider and oscillates with it. At what points in the motion is the friction force on the coin greatest? The least? Justify your answers.

Q14.7 Two identical gliders on an air track are connected by an ideal spring. Could such a system undergo SHM? Explain. How would the period compare with that of a single glider attached to a spring whose other end is rigidly attached to a stationary object? Explain.

Q14.8 You are captured by Martians, taken into their ship, and put to sleep. You awake some time later and find yourself locked in a small room with no windows. All the Martians have left you with is your digital watch, your school ring, and your long silver-chain necklace. Explain how you can determine whether you are still on earth or have been transported to Mars.

Q14.9 The system shown in Fig. 14.17 is mounted in a lift. What happens to the period of the motion (does it increase, decrease, or remain the same) if the lift (a) accelerates upward at $5.0 \ \text{m/s}^2$; (b) moves upward at a steady $5.0 \ \text{m/s}$; (c) accelerates downward at $5.0 \ \text{m/s}^2$? Justify your answers.

Q14.10 If a pendulum has a period of 2.5 s on earth, what would be its period in a space station orbiting the earth? If a mass hung from a vertical spring has a period of 5.0 s on earth, what would its period be in the space station? Justify your answers.

Q14.11 A simple pendulum is mounted in a lift. What happens to the period of the pendulum (does it increase, decrease, or remain the same) if the lift (a) accelerates upward at $5.0 \ \text{m/s}^2$; (b) moves upward at a steady $5.0 \ \text{m/s}$; (c) accelerates downward at $5.0 \ \text{m/s}^2$; (d) accelerates downward at $9.8 \ \text{m/s}^2$? Justify your answers.

Q14.12 What should you do to the length of the string of a simple pendulum to (a) double its frequency; (b) double its period; (c) double its angular frequency?

Q14.13 If a pendulum clock is taken to a mountaintop, does it gain or lose time, assuming it is correct at a lower elevation? Explain.

Q14.14 When the amplitude of a simple pendulum increases, should its period increase or decrease? Give a qualitative argument; do not rely on Eq. (14.35). Is your argument also valid for a physical pendulum?

Q14.15 Why do short dogs (like Chihuahuas) walk with quicker strides than do tall dogs (like Great Danes)?

Q14.16 At what point in the motion of a simple pendulum is the string tension greatest? Least? In each case give the reasoning behind your answer.

Q14.17 Could a standard of time be based on the period of a certain standard pendulum? What advantages and disadvantages would such a standard have compared to the actual present-day standard discussed in Section 1.3?

Q14.18 For a simple pendulum, clearly distinguish between ω (the angular speed) and ω (the angular frequency). Which is constant and which is variable?

Q14.19 In designing structures in an earthquake-prone region, how should the natural frequencies of oscillation of a structure relate to typical earthquake frequencies? Why? Should the structure have a large or small amount of damping?

EXERCISES

Section 14.1 Describing Oscillation

14.1 • **BIO** (a) **Music.** When a person sings, his or her vocal cords vibrate in a repetitive pattern that has the same frequency as the note that is sung. If someone sings the note B flat, which has a frequency of 466 Hz, how much time does it take the person's vocal cords to vibrate through one complete cycle, and what is the angular frequency of the cords? (b) **Hearing.** When sound waves strike the eardrum, this membrane vibrates with the same frequency as the sound. The highest pitch that young humans can hear has a period of $50.0 \ \mu\text{s}$. What are the frequency and angular frequency of the vibrating eardrum for this sound? (c) **Vision.** When light having vibrations with angular frequency ranging from $2.7 \times 10^{15} \ \text{rad/s}$ to $4.7 \times 10^{15} \ \text{rad/s}$ strikes the retina of the eye, it stimulates the receptor cells there and is perceived as visible light. What are the limits of the period and frequency of this light? (d) **Ultrasound.** High-frequency sound waves (ultrasound) are used to probe the interior of the body, much as x rays do. To detect small objects such as tumors, a frequency of around 4.80 MHz is used. What are the period and angular frequency of the molecular vibrations caused by this pulse of sound?
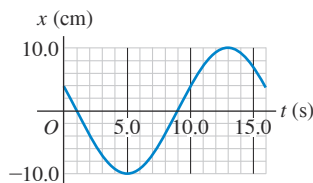
14.2 • If an object on a horizontal, frictionless surface is attached to a spring, displaced, and then released, it will oscillate. If it is displaced 0.115 m from its equilibrium position and released with zero initial speed, then after 0.815 s its displacement is found to be 0.115 m on the opposite side, and it has passed the equilibrium position once during this interval. Find (a) the amplitude; (b) the period; (c) the frequency.

14.3 • The tip of a tuning fork goes through 440 complete vibrations in 0.500 s. Find the angular frequency and the period of the motion.

14.4 • The displacement of an oscillating object as a function of time is shown in **Fig. E14.4.** What are (a) the frequency; (b) the amplitude; (c) the period; (d) the angular frequency of this motion?

14.5 •• A machine part is undergoing SHM with a frequency of 5.10 Hz and amplitude 1.80 cm. How long does it take the part to go from $x = 0$ to $x = -1.80$ cm?
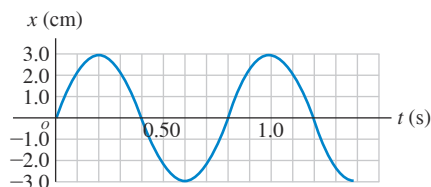
Figure **E14.4**

Section 14.2 Simple Harmonic Motion

14.6 • You are pushing your niece on a playground swing. The swing seat is suspended from a horizontal bar by two light chains. Based on your experience with swings, estimate the length of each chain. Treat the motion of the child as that of a simple pendulum and assume that for safety the amplitude of the motion is kept small. You give your niece a light push each time she reaches her closest distance from you. How much time elapses between your pushes?

14.7 • A 2.40 kg ball is attached to an unknown spring and allowed to oscillate. **Figure E14.7** shows a graph of the ball's position x as a function of time t. What are the oscillation's (a) period, (b) frequency, (c) angular frequency, and (d) amplitude? (e) What is the force constant of the spring?

Figure **E14.7**

14.8 •• In a physics lab, you attach a 0.200 kg air-track glider to the end of an ideal spring of negligible mass and start it oscillating. The elapsed time from when the glider first moves through the equilibrium point to the second time it moves through that point is 2.60 s. Find the spring's force constant.

14.9 • When a body of unknown mass is attached to an ideal spring with force constant 128 N/m, it is found to vibrate with a frequency of 6.20 Hz. Find (a) the period of the motion; (b) the angular frequency; (c) the mass of the body.

14.10 • When a 0.800 kg mass oscillates on an ideal spring, the frequency is 1.41 Hz. What will the frequency be if 0.260 kg are (a) added to the original mass and (b) subtracted from the original mass? Try to solve this problem *without* finding the force constant of the spring.

14.11 •• An object is undergoing SHM with period 0.820 s and amplitude 0.320 m. At $t = 0$ the object is at $x = 0.320$ m and is instantaneously at rest. Calculate the time it takes the object to go (a) from $x = 0.320$ m to $x = 0.160$ m and (b) from $x = 0.160$ m to $x = 0$.

14.12 • A small block is attached to an ideal spring and is moving in SHM on a horizontal, frictionless surface. When the block is at $x = 0.310$ m, the acceleration of the block is -5.96 m/s². What is the frequency of the motion?

14.13 • A 2.00 kg, frictionless block is attached to an ideal spring with force constant 300 N/m. At $t = 0$ the spring is neither stretched nor compressed and the block is moving in the negative direction at 12.0 m/s. Find (a) the amplitude and (b) the phase angle. (c) Write an equation for the position as a function of time.

14.14 •• Repeat Exercise 14.13, but assume that at $t = 0$ the block has velocity -4.00 m/s and displacement $+0.200$ m away from equilibrium.

14.15 •• A block of mass m is undergoing SHM on a horizontal, frictionless surface while attached to a light, horizontal spring. The spring has force constant k, and the amplitude of the SHM is A. The block has $v = 0$, and $x = +A$ at $t = 0$. It first reaches $x = 0$ when $t = T/4$, where T is the period of the motion. (a) In terms of T, what is the time t when the block first reaches $x = A/2$? (b) The block has its maximum speed when $t = T/4$. What is the value of t when the speed of the block first reaches the value $v_{max}/2$? (c) Does $v = v_{max}/2$ when $x = A/2$?
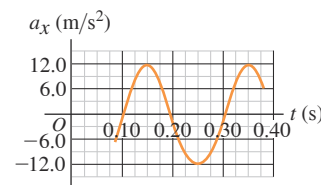
14.16 •• A small block is attached to an ideal spring and is moving in SHM on a horizontal, frictionless surface. When the amplitude of the motion is 0.090 m, it takes the block 2.70 s to travel from $x = 0.090$ m to $x = -0.090$ m. If the amplitude is doubled, to 0.180 m, how long does it take the block to travel (a) from $x = 0.180$ m to $x = -0.180$ m and (b) from $x = 0.090$ m to $x = -0.090$ m?

14.17 • **BIO** **Weighing Astronauts.** This procedure has been used to "weigh" astronauts in space: A 42.5 kg chair is attached to a spring and allowed to oscillate. When it is empty, the chair takes 1.30 s to make one complete vibration. But with an astronaut sitting in it, with her feet off the floor, the chair takes 2.54 s for one cycle. What is the mass of the astronaut?

14.18 • A 0.450 kg object undergoing SHM has $a_x = -2.50$ m/s² when $x = 0.400$ m. What is the time for one oscillation?

14.19 • On a frictionless, horizontal air track, a glider oscillates at the end of an ideal spring of force constant 2.30 N/cm. The graph in **Fig. E14.19** shows the acceleration of the glider as a function of time. Find (a) the mass of the glider; (b) the maximum displacement of the glider from the equilibrium point; (c) the maximum force the spring exerts on the glider.

Figure **E14.19**

14.20 • A 0.500 kg mass on a spring has velocity as a function of time given by $v_x(t) = -(3.60 \text{ cm/s}) \sin[(4.71 \text{ rad/s})t - (\pi/2)]$. What are (a) the period; (b) the amplitude; (c) the maximum acceleration of the mass; (d) the force constant of the spring?

14.21 • A 2.20-kg mass on a spring has displacement as a function of time given by

$$x(t) = (7.40 \text{ cm}) \cos[(4.16 \text{ rad/s})t - 2.42]$$

Find (a) the time for one complete vibration; (b) the force constant of the spring; (c) the maximum speed of the mass; (d) the maximum force on the mass; (e) the position, speed, and acceleration of the mass at $t = 1.00$ s; (f) the force on the mass at that time.

14.22 • **BIO** **Weighing a Virus.** In February 2004, scientists at Purdue University used a highly sensitive technique to measure the mass of a vaccinia virus (the kind used in smallpox vaccine). The procedure involved measuring the frequency of oscillation of a tiny sliver of silicon (just 28 nm long) with a laser, first without the virus and then after the virus had attached itself to the silicon. The difference in mass caused a change in the frequency. We can model such a process as a mass on a spring. (a) Show that the ratio of the frequency with the virus attached (f_{S+V}) to the frequency without the virus (f_S) is given by $f_{S+V}/f_S = 1/\sqrt{1 + (m_V/m_S)}$, where m_V is the mass of the virus and m_S is the mass of the silicon sliver. Notice that it is *not* necessary to know or measure the force constant of the spring. (b) In some data, the silicon sliver has a mass of 2.12×10^{-16} g and a frequency of 2.04×10^{15} Hz without the virus and 2.86×10^{14} Hz with the virus. What is the mass of the virus, in grams and in femtograms?

14.23 •• **CALC** The *jerk* is defined to be the time rate of change of the acceleration. (a) If the velocity of an object undergoing SHM is given by $v_x = -\omega A \sin(\omega t)$, what is the equation for the x-component of the jerk as a function of time? (b) What is the value of x for the object when the x-component of the jerk has its largest positive value? (c) What is x when the x-component of the jerk is most negative? (d) When it is zero? (e) If v_x equals -0.040 s^2 times the x-component of the jerk for all t, what is the period of the motion?

Section 14.3 Energy in Simple Harmonic Motion

14.24 •• For the oscillating object in Fig. E14.4, what are (a) its maximum speed and (b) its maximum acceleration?

14.25 • A small block is attached to an ideal spring and is moving in SHM on a horizontal, frictionless surface. The amplitude of the motion is 0.165 m. The maximum speed of the block is 3.90 m/s. What is the maximum magnitude of the acceleration of the block?

14.26 • A small block is attached to an ideal spring and is moving in SHM on a horizontal, frictionless surface. The amplitude of the motion is 0.200 m and the period is 3.45 s. What are the speed and acceleration of the block when $x = 0.160$ m?

14.27 • A 0.155 kg toy is undergoing SHM on the end of a horizontal spring with force constant $k = 305$ N/m. When the toy is 1.25×10^{-2} m from its equilibrium position, it is observed to have a speed of 0.295 m/s. What are the toy's (a) total energy at any point of its motion; (b) amplitude of motion; (c) maximum speed during its motion?

14.28 •• A harmonic oscillator has angular frequency ω and amplitude A. (a) What are the magnitudes of the displacement and velocity when the elastic potential energy is equal to the kinetic energy? (Assume that $U = 0$ at equilibrium.) (b) How often does this occur in each cycle? What is the time between occurrences? (c) At an instant when the displacement is equal to $A/2$, what fraction of the total energy of the system is kinetic and what fraction is potential?

14.29 • A 0.500 kg glider, attached to the end of an ideal spring with force constant $k = 450$ N/m, undergoes SHM with an amplitude of 0.040 m. Compute (a) the maximum speed of the glider; (b) the speed of the glider when it is at $x = -0.015$ m; (c) the magnitude of the maximum acceleration of the glider; (d) the acceleration of the glider at $x = -0.015$ m; (e) the total mechanical energy of the glider at any point in its motion.

14.30 •• A block of mass m is undergoing SHM on a horizontal, frictionless surface while attached to a light, horizontal spring. The spring has force constant k, and the amplitude of the motion of the block is A. (a) The average speed is the total distance traveled by the block divided by the time it takes it to travel this distance. Calculate the average speed for one cycle of the SHM. (b) How does the average speed for one cycle compare to the maximum speed v_{max}? (c) Is the average speed more or less than half the

maximum speed? Based on your answer, does the block spend more time while traveling at speeds greater than $v_{max}/2$ or less than $v_{max}/2$?

14.31 •• A block of mass m is undergoing SHM on a horizontal, frictionless surface while it is attached to a light, horizontal spring that has force constant k. The amplitude of the SHM of the block is A. What is the distance $|x|$ of the block from its equilibrium position when its speed v is half its maximum speed v_{max}? Is this distance larger or smaller than $A/2$?

14.32 •• A block with mass $m = 0.300$ kg is attached to one end of an ideal spring and moves on a horizontal frictionless surface. The other end of the spring is attached to a wall. When the block is at $x = +0.240$ m, its acceleration is $a_x = -12.0$ m/s^2 and its velocity is $v_x = +4.00$ m/s. What are (a) the spring's force constant k; (b) the amplitude of the motion; (c) the maximum speed of the block during its motion; and (d) the maximum magnitude of the block's acceleration during its motion?

14.33 •• You are watching an object that is moving in SHM. When the object is displaced 0.600 m to the right of its equilibrium position, it has a velocity of 2.20 m/s to the right and an acceleration of 8.40 m/s^2 to the left. How much farther from this point will the object move before it stops momentarily and then starts to move back to the left?

14.34 •• A mass is oscillating with amplitude A at the end of a spring. How far (in terms of A) is this mass from the equilibrium position of the spring when the elastic potential energy equals the kinetic energy?

14.35 • A 2.00 kg frictionless block attached to an ideal spring with force constant 315 N/m is undergoing simple harmonic motion. When the block has displacement $+0.200$ m, it is moving in the negative x-direction with a speed of 4.00 m/s. Find (a) the amplitude of the motion; (b) the block's maximum acceleration; and (c) the maximum force the spring exerts on the block.

Section 14.4 Applications of Simple Harmonic Motion

14.36 • A proud deep-sea fisherman hangs a 65.0 kg fish from an ideal spring having negligible mass. The fish stretches the spring 0.180 m. (a) Find the force constant of the spring. The fish is now pulled down 5.00 cm and released. (b) What is the period of oscillation of the fish? (c) What is the maximum speed it will reach?

14.37 • A 180 g glider on a horizontal, frictionless air track is attached to a fixed ideal spring with force constant 180 N/m. At the instant you make measurements on the glider, it is moving at 0.765 m/s and is 2.50 cm from its equilibrium point. Use *energy conservation* to find (a) the amplitude of the motion and (b) the maximum speed of the glider. (c) What is the angular frequency of the oscillations?

14.38 •• A uniform, solid metal disk of mass 6.90 kg and diameter 21.0 cm hangs in a horizontal plane, supported at its center by a vertical metal wire. You find that it requires a horizontal force of 4.25 N tangent to the rim of the disk to turn it by 3.37°, thus twisting the wire. You now remove this force and release the disk from rest. (a) What is the torsion constant for the metal wire? (b) What are the frequency and period of the torsional oscillations of the disk? (c) Write the equation of motion for $\theta(t)$ for the disk.

14.39 • A thrill-seeking cat with mass 4.00 kg is attached by a harness to an ideal spring of negligible mass and oscillates vertically in SHM. The amplitude is 0.050 m, and at the highest point of the motion the spring has its natural unstretched length. Calculate the elastic potential energy of the spring (take it to be zero for the unstretched spring), the kinetic energy of the cat, the gravitational potential energy of the system relative to the lowest point of the motion, and the sum of these three energies when the cat is (a) at its highest point; (b) at its lowest point; (c) at its equilibrium position.