

# Corporate Finance 

FOURTH EDITION
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| A | market value of assets, premerger | $P_{i}$ | price of security $i$ |
| :---: | :---: | :---: | :---: |
|  | total value of acquirer | P/E | price-earnings ratio |
| APR | annual percentage rate | PMT | annuity spreadsheet notation |
| $B$ | risk-free investment |  | for cash flow |
|  | in the replicating portfolio | PV | present value; annuity spreadsheet |
| C | cash flow, call option price |  | notation for the initial amount |
| $\operatorname{Corr}\left(R_{i}, R_{j}\right)$ | correlation between returns of $i$ and $j$ | $q$ | dividend yield |
| $\operatorname{Cov}\left(R_{i}, R_{j}\right)$ | covariance between returns of $i$ and $j$ | $p$ | risk-neutral probability |
| CPN | coupon payment | $r$ | interest rate, discount rate of cost |
| D | market value of debt |  | of capital |
| $d$ | debt-to-value ratio | $R_{i}$ | return of security $i$ |
| Div ${ }_{t}$ | dividends paid in year $t$ | $R_{m k t}$ | return of the market portfolio |
| dis | discount from face value | $R_{P}$ | return on portfolio $P$ |
| E | market value of equity | RATE | annuity spreadsheet notation |
| EAR | effective annual rate |  |  |
| EBIT | earnings before interest and taxes | $r_{E}, r_{D}$ | equity and debt costs of capital |
| EBITDA | earnings before interest, taxes, | $r_{f}$ | risk-free interest rate |
|  | depreciation, and amortization | $r_{i}$ | required return or cost of capital of security $i$ |
| $E P S_{t}$ | earnings per share on date $t$ |  |  |
| $E\left[R_{i}\right]$ | expected return of security $i$ | $r_{U}$ | unlevered cost of capital |
| $F_{\text {F }} F_{T}$ | one-year and $T$-year forward exchange rate | $\begin{aligned} & r_{\text {wacc }} \\ & S \end{aligned}$ | weighted average cost of capital stock price, spot exchange rate, value of all synergies |
| $F C F_{t}$ | free cash flow at date $t$ |  |  |
| FV | future value, face value of a bond | $S D\left(R_{i}\right)$ | standard deviation (volatility) of return of security $i$ |
| $g$ | growth rate | $T$ | option expiration date, maturity |
| I | initial investment or initial capital committed to the project |  | market value of target |
| Int ${ }_{\text {t }}$ | interest expense on date $t$ | U | market value of unlevered equity |
| IRR | internal rate of return |  | enterprise value on date $t$ |
| K | strike price | $\operatorname{Var}(R)$ | variance of return $R$. |
| k | interest coverage ratio, compounding periods per year | $\begin{aligned} & x_{i} \\ & Y T C \end{aligned}$ | portfolio weight of investment in $i$ yield to call on a callable bond |
| $L$ | lease payment, market value of liabilities | YTM | yield to maturity |
| 1 n | natural logarithm | $\alpha_{i}$ | alpha of security $i$ |
| $M V_{i}$ | total market capitalization of security $i$ | $\beta_{D,} \beta_{E}$ | beta of debt or equity |
| $N{ }^{i}$ | number of cash flows, terminal date, | $\beta_{i}$ | beta of security $i$ with respect to the market portfolio |
| $N_{i}$ | number of shares outstanding of security $i$ | $\beta_{s}^{P}$ | beta of security $i$ with respect to portfolio $P$ |
| NPER |  | $\beta_{U}$ | beta of unlevered firm |
|  | for the number of periods or dates of the last cash flow | $\Delta$ | shares of stock in the replicating portfolio; sensitivity of option price |
| NPV | net present value |  | to stock price |
| $P$ | price, initial principal or deposit, | $\sigma$ | volatility |
|  | or equivalent present value, | $\tau$ | tax rate |
|  | put option price | $\tau_{c}$ | marginal corporate tax rate |

# 10.1 Risk and Return: Insights from 89 Years of Investor History 

We begin our look at risk and return by illustrating how risk affects investor decisions and returns. Suppose your great-grandparents invested $\$ 100$ on your behalf at the end of 1925 . They instructed their broker to reinvest any dividends or interest earned in the account until the beginning of 2015 . How would that $\$ 100$ have grown if it were placed in one of the following investments?

1. Standard \& Poor's 500 (S\&P 500): A portfolio, constructed by Standard and Poor's, comprising 90 U.S. stocks up to 1957 and 500 U.S. stocks after that. The firms represented are leaders in their respective industries and are among the largest firms, in terms of market value, traded on U.S. markets.
2. Small Stocks: A portfolio, updated quarterly, of U.S. stocks traded on the NYSE with market capitalizations in the bottom $20 \%$.
3. World Portfolio: A portfolio of international stocks from all of the world's major stock markets in North America, Europe, and Asia. ${ }^{1}$
4. Corporate Bonds: A portfolio of long-term, AAA-rated U.S. corporate bonds with maturities of approximately 20 years. ${ }^{2}$
5. Treasury Bills: An investment in one-month U.S. Treasury bills.

Figure 10.1 shows the result, through the start of 2015, of investing $\$ 100$ at the end of 1925 in each of these five investment portfolios, ignoring transactions costs. During this 89-year period in the United States, small stocks experienced the highest long-term return, followed by the large stocks in the S\&P 500, the international stocks in the world portfolio, corporate bonds, and finally Treasury bills. All of the investments grew faster than inflation, as measured by the consumer price index (CPI).

At first glance the graph is striking-had your great-grandparents invested \$100 in the small stock portfolio, the investment would be worth more than $\$ 4.6$ million at the beginning of 2015! By contrast, if they had invested in Treasury bills, the investment would be worth only about $\$ 2,000$. Given this wide difference, why invest in anything other than small stocks?

But first impressions can be misleading. While over the full horizon stocks (especially small stocks) did outperform the other investments, they also endured periods of significant losses. Had your great-grandparents put the $\$ 100$ in a small stock portfolio during the Depression era of the 1930s, it would have grown to $\$ 181$ in 1928 , but then fallen to only $\$ 15$ by 1932. Indeed, it would take until World War II for stock investments to outperform corporate bonds.

Even more importantly, your great-grandparents would have sustained losses at a time when they likely needed their savings the most-in the depths of the Great Depression. A similar story held during the 2008 financial crisis: All of the stock portfolios declined by more than $50 \%$, with the small stock portfolio declining by almost $70 \%$ (over $\$ 1.5$ million!) from its peak in 2007 to its lowest point in 2009. Again, many investors faced a double whammy: an increased risk of being unemployed (as firms started laying off employees)

[^0]

The chart shows the growth in value of $\$ 100$ invested in 1925 if it were invested in U.S. large stocks, small stocks, world stocks, corporate bonds, or Treasury bills, with the level of the consumer price index (CPI) shown as a reference. Returns were calculated at year-end assuming all dividends and interest are reinvested and excluding transactions costs. Note that while stocks have generally outperformed bonds and bills, they have also endured periods of significant losses (numbers shown represent peak to trough decline, with the decline in small stocks in red and the S\&P 500 in blue).
Source: Chicago Center for Research in Security Prices, Standard and Poor's, MSCI, and Global Financial Data.
precisely when the value of their savings eroded. Thus, while the stock portfolios had the best performance over this 89 -year period, that performance came at a cost-the risk of large losses in a downturn. On the other hand, Treasury bills enjoyed steady-albeit modestgains each year.

Few people ever make an investment for 89 years, as depicted in Figure 10.1. To gain additional perspective on the risk and return of these investments, Figure 10.2 shows the results for more realistic investment horizons and different initial investment dates. Panel (a), for example, shows the value of each investment after one year and illustrates that if we rank the investments by the volatility of their annual increases and decreases in value, we obtain the same ranking we observed with regard to performance: Small stocks had the most variable returns, followed by the S\&P 500, the world portfolio, corporate bonds, and finally Treasury bills.

Panels (b), (c), and (d) of Figure 10.2 show the results for 5 -, 10 -, and 20 -year investment horizons, respectively. Note that as the horizon lengthens, the relative performance of the stock portfolios improves. That said, even with a 10 -year horizon there were periods during which stocks underperformed Treasuries. And while investors in small stocks most often came out ahead, this was not assured even with a 20 -year horizon: For investors in

FIGURE 10.2
— Small Stocks — S\&P 500 —Corporate Bonds —Treasury Bills


Each panel shows the result of investing $\$ 100$ at the end of the initial investment year, in each investment opportunity, for horizons of $1,5,10$, or 20 years. That is, each point on the plot is the result of an investment over the specified horizon, plotted as a function of the initial investment date. Dividends and interest are reinvested and transaction costs are excluded. Note that small stocks show the greatest variation in performance at the one-year horizon, followed by large stocks and then corporate bonds. For longer horizons, the relative performance of stocks improved, but they remained riskier.
Source Data: Chicago Center for Research in Security Prices, Standard and Poor's, MSCI, and Global Financial Data.
the early 1980s, small stocks did worse than both the S\&P 500 and corporate bonds over the subsequent 20 years. Finally, stock investors with long potential horizons might find themselves in need of cash in intervening years, and be forced to liquidate at a loss relative to safer alternatives.

In Chapter 3, we explained why investors are averse to fluctuations in the value of their investments, and that investments that are more likely to suffer losses in downturns must compensate investors for this risk with higher expected returns. Figures 10.1 and 10.2 provide compelling historical evidence of this relationship between risk and return, just as we should expect in an efficient market. But while it is clear that investors do not like risk and thus demand a risk premium to bear it, our goal in this chapter is to quantify this relationship. We want to explain how much investors demand (in terms of a higher expected return) to bear a given level of risk. To do so, we must first develop tools that will allow us to measure risk and return - the objective of the next section.

CONCEPT CHECK 1. For an investment horizon from 1926 to 2012, which of the following investments had the highest return: the S\&P 500, small stocks, world portfolio, corporate bonds, or Treasury bills? Which had the lowest return?
2. For an investment horizon of just one year, which of these investments was the most variable? Which was the least variable?

### 10.2 Common Measures of Risk and Return

When a manager makes an investment decision or an investor purchases a security, they have some view as to the risk involved and the likely return the investment will earn. Thus, we begin our discussion by reviewing the standard ways to define and measure risks.

## Probability Distributions

Different securities have different initial prices, pay different cash flows, and sell for different future amounts. To make them comparable, we express their performance in terms of their returns. The return indicates the percentage increase in the value of an investment per dollar initially invested in the security. When an investment is risky, there are different returns it may earn. Each possible return has some likelihood of occurring. We summarize this information with a probability distribution, which assigns a probability, $p_{R}$, that each possible return, $R$, will occur.

Let's consider a simple example. Suppose BFI stock currently trades for $\$ 100$ per share. You believe that in one year there is a $25 \%$ chance the share price will be $\$ 140$, a $50 \%$ chance it will be $\$ 110$, and a $25 \%$ chance it will be $\$ 80$. BFI pays no dividends, so these payoffs correspond to returns of $40 \%, 10 \%$, and $-20 \%$, respectively. Table 10.1 summarizes the probability distribution for BFI's returns.

We can also represent the probability distribution with a histogram, as shown in Figure 10.3.

## Expected Return

Given the probability distribution of returns, we can compute the expected return. We calculate the expected (or mean) return as a weighted average of the possible returns, where the weights correspond to the probabilities. ${ }^{3}$

Expected (Mean) Return

$$
\begin{equation*}
\text { Expected Return }=E[R]=\sum_{R} p_{R} \times R \tag{10.1}
\end{equation*}
$$

## TABLE 10.1 Probability Distribution of Returns for BFI

|  |  | Probability Distribution |  |
| :---: | :---: | :---: | :---: |
| Current Stock Price (\$) | Stock Price in One Year (\$) | Return, $R$ | Probability, $\boldsymbol{p}_{\boldsymbol{R}}$ |
| 100 | 140 | 0.40 | $25 \%$ |
|  | 110 | 0.10 | $50 \%$ |
|  | 80 | -0.20 | $25 \%$ |

[^1]
## FIGURE 10.3

Probability Distribution of Returns for BFI
The height of a bar in the histogram indicates the likelihood of the associated outcome.


The expected return is the return we would earn on average if we could repeat the investment many times, drawing the return from the same distribution each time. In terms of the histogram, the expected return is the "balancing point" of the distribution, if we think of the probabilities as weights. The expected return for BFI is

$$
E\left[R_{B F I}\right]=25 \%(-0.20)+50 \%(0.10)+25 \%(0.40)=10 \%
$$

This expected return corresponds to the balancing point in Figure 10.3.

## Variance and Standard Deviation

Two common measures of the risk of a probability distribution are its variance and standard deviation. The variance is the expected squared deviation from the mean, and the standard deviation is the square root of the variance.

Variance and Standard Deviation of the Return Distribution

$$
\begin{align*}
\operatorname{Var}(R) & =E\left[(R-E[R])^{2}\right]=\sum_{R} p_{R} \times(R-E[R])^{2} \\
S D(R) & =\sqrt{\operatorname{Var}(R)} \tag{10.2}
\end{align*}
$$

If the return is risk-free and never deviates from its mean, the variance is zero. Otherwise, the variance increases with the magnitude of the deviations from the mean. Therefore, the variance is a measure of how "spread out" the distribution of the return is. The variance of BFI's return is

$$
\begin{aligned}
\operatorname{Var}\left(R_{B F I}\right) & =25 \% \times(-0.20-0.10)^{2}+50 \% \times(0.10-0.10)^{2}+25 \% \times(0.40-0.10)^{2} \\
& =0.045
\end{aligned}
$$

The standard deviation of the return is the square root of the variance, so for BFI,

$$
\begin{equation*}
S D(R)=\sqrt{\operatorname{Var}(R)}=\sqrt{0.045}=21.2 \% \tag{10.3}
\end{equation*}
$$


[^0]:    ${ }^{1}$ Based on a World Market Index constructed by Global Financial Data, with approximate initial weights of $44 \%$ North America, $44 \%$ Europe, and $12 \%$ Asia, Africa, and Australia.
    ${ }^{2}$ Based on Global Financial Data's Corporate Bond Index.

[^1]:    ${ }^{3}$ The notation $\Sigma_{R}$ means that we calculate the sum of the expression (in this case, $p_{R} \times R$ ) over all possible returns $R$.

