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Technology Update

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SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Set the net force acting on the plank equal to zero: $F_1 + F_2 - mg = 0$
2. Set the net torque acting on the plank equal to zero: $-F_1(L) + mg\left(\frac{1}{4}L\right) = 0$
3. Note that the torque condition involves only one of the two unknowns, F_1. Use this condition to solve for F_1: $F_1 = \frac{1}{4}mg$
4. Substitute F_1 into the force condition to solve for F_2: $F_2 = mg - \frac{1}{4}mg = \frac{3}{4}mg$

INSIGHT

As expected, the results are identical to those obtained previously. Note that in this case the torque produced by the child would cause a counterclockwise rotation, hence it is positive. Thus, the magnitude *and* sign of the torque produced by a given force depend on the location chosen for the axis of rotation.

YOUR TURN

Suppose the child moves to a new position, with the result that the force exerted by the father is reduced to $0.60mg$. Did the child move to the left or to the right? How far did the child move?

A third choice for the axis of rotation is considered in Problem 24. As expected, all three choices give the same results.

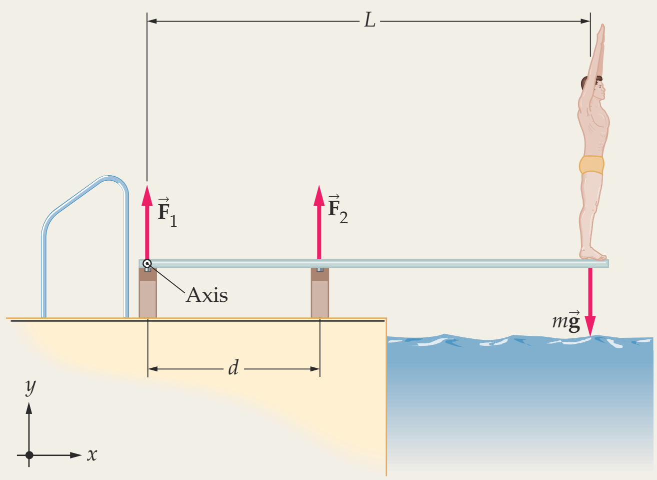
In the next Example, we show that the forces supporting a person or other object sometimes act in different directions. To emphasize the direction of the forces, we solve the Example in terms of the components of the relevant forces.

EXAMPLE 4 TAKING THE PLUNGE

A 5.00-m-long diving board of negligible mass is supported by two pillars. One pillar is at the left end of the diving board, as shown below; the other is 1.50 m away. Find the forces exerted by the pillars when a 90.0-kg diver stands at the far end of the board.

PICTURE THE PROBLEM

We choose upward to be the positive direction for the forces. When calculating torques, we use the left end of the diving board as the axis of rotation. Note that $\vec{\mathbf{F}}_2$ would cause a counterclockwise rotation if it acted alone, so its torque is positive. On the other hand, $m\vec{\mathbf{g}}$ would cause a clockwise rotation, so its torque is negative. Finally, $\vec{\mathbf{F}}_2$ acts at a distance d from the axis of rotation, and $m\vec{\mathbf{g}}$ acts at a distance L.



STRATEGY

As usual in static equilibrium problems, we use the conditions of (i) zero net force and (ii) zero net torque to determine the unknown forces, $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$. In this system all forces act in the positive or negative y direction; thus we need only set the net y component of force equal to zero.

SOLUTION

1. Set the net y component of force acting on the diving board equal to zero: $\sum F_y = F_{1,y} + F_{2,y} - mg = 0$
2. Calculate the torque due to each force, using the left end of the board as the axis of rotation. Note that each force is at right angles to the radius and that $\vec{\mathbf{F}}_1$ goes directly through the axis of rotation:

$$\tau_1 = F_{1,y}(0) = 0$$
$$\tau_2 = F_{2,y}(d)$$
$$\tau_3 = -mg(L)$$

3. Set the net torque acting on the diving board equal to zero: $\sum \tau = F_{1,y}(0) + F_{2,y}(d) - mg(L) = 0$
4. Solve the torque equation for the force $F_{2,y}$:

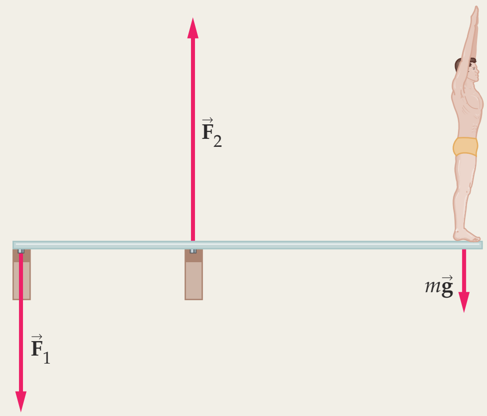
$$F_{2,y} = mg(L/d)$$
$$= (90.0\ \text{kg})(9.81\ \text{m/s}^2)(5.00\ \text{m}/1.50\ \text{m}) = 2940\ \text{N}$$

5. Use the force equation to determine $F_{1,y}$:

$$F_{1,y} = mg - F_{2,y}$$
$$= (90.0\ \text{kg})(9.81\ \text{m/s}^2) - 2940\ \text{N} = -2060\ \text{N}$$

INSIGHT

The first point to notice about our solution is that $F_{1,y}$ is negative, which means that $\vec{\mathbf{F}}_1$ is actually directed *downward*, as shown to the right. To see why, imagine for a moment that the board is no longer connected to the first pillar. In this case, the board would rotate clockwise about the second pillar, and the left end of the board would move upward. Thus, a downward force is required on the left end of the board to hold it in place.



The second point is that both pillars exert forces with magnitudes that are considerably larger than the diver's weight, $mg = 883$ N. In particular, the first pillar must pull downward with a force of $2.33mg$, while the second pillar pushes upward with a force of $2.33mg + mg = 3.33mg$. This is not unusual. In fact, it is common for the forces in a structure, such as a bridge, a building, or the human body, to be much greater than the weight it supports.

PRACTICE PROBLEM

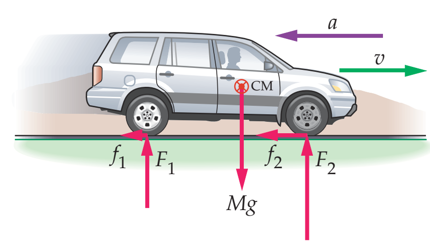
Find the forces exerted by the pillars when the diver is 1.00 m from the right end. [**Answer:** $F_{1,y} = -1470$ N, $F_{2,y} = 2350$ N]

Some related homework problems: Problem 26, Problem 32

To this point we have ignored the mass of the plank holding the child and the diving board holding the swimmer, since they were described as lightweight. If we want to consider the torque exerted by an extended object of finite mass, however, we can simply treat it as if all its mass were concentrated at its center of mass. We consider such a system in the next Active Example.

REAL-WORLD PHYSICS

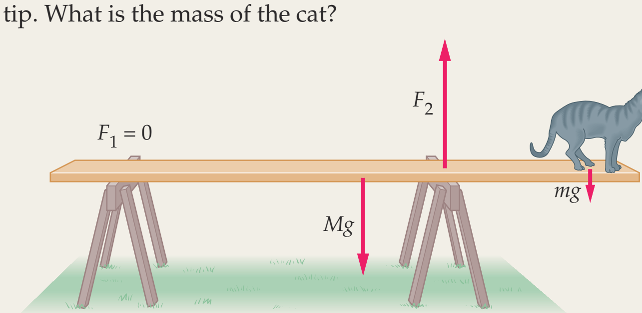
Applying the brakes



▲ As the brakes are applied on this SUV, rotational equilibrium demands that the normal forces exerted on the front tires be greater than the normal forces exerted on the rear tires—which is why braking cars are "nose down" during a rapid stop. For this reason, many cars use disk brakes for the front wheels and less powerful drum brakes for the rear wheels. As the disk brakes wear, they tend to coat the front wheels with dust from the brake pads, which give the front wheels a characteristic "dirty" look.

ACTIVE EXAMPLE 2 WALKING THE PLANK: FIND THE MASS

A cat walks along a uniform plank that is 4.00 m long and has a mass of 7.00 kg. The plank is supported by two sawhorses, one 0.440 m from the left end of the board and the other 1.50 m from its right end. When the cat reaches the right end, the plank just begins to tip. What is the mass of the cat?



SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Since the board is just beginning to tip, there is no weight on the left sawhorse: $F_1 = 0$

2. Calculate the torque about the right sawhorse: $Mg(0.500\ \text{m}) - mg(1.50\ \text{m}) = 0$

3. Solve the torque equation for the mass of the cat, m: $m = 0.333M = 2.33$ kg

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INSIGHT

Note that we did not include a torque for the left sawhorse, since F_1 is zero. As an exercise, you might try repeating the calculation with the axis of rotation at the left sawhorse, or at the center of mass of the plank.

YOUR TURN

Write both the zero force and zero torque conditions for the case where the axis of rotation is at the left sawhorse.

Forces with Both Vertical and Horizontal Components

Note that all of the previous examples have dealt with forces that point either directly upward or directly downward. We now consider a more general situation, where forces may have both vertical and horizontal components. For example, consider the wall-mounted lamp (sconce) shown in **Figure 6**. The sconce consists of a light curved rod that is bolted to the wall at its lower end. Suspended from the upper end of the rod, a horizontal distance H from the wall, is the lamp of mass m. The rod is also connected to the wall by a horizontal wire a vertical distance V above the bottom of the rod.

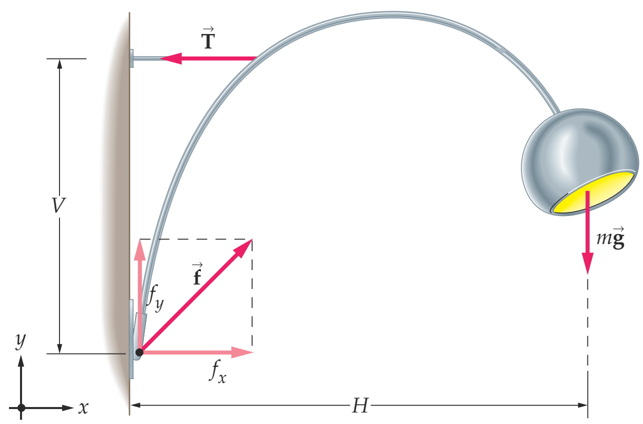

FIGURE 6 A lamp in static equilibrium

A wall-mounted lamp of mass m is suspended from a light curved rod. The bottom of the rod is bolted to the wall. The rod is also connected to the wall by a horizontal wire a vertical distance V above the bottom of the rod.

Now, suppose we are designing this sconce to be placed in the lobby of a building on campus. To ensure its structural stability, we would like to know the tension T the wire must exert and the vertical and horizontal components of the force $\vec{f}$ that must be exerted by the bolt on the rod. This information will be important in deciding on the type of wire and bolt to be used in the construction.

To find these forces, we apply the same conditions as before: the net force and the net torque must be zero. In this case, however, forces may have both horizontal and vertical components. Thus, the condition of zero net force is really two separate conditions: (i) zero net force in the horizontal direction; and (ii) zero net force in the vertical direction. These two conditions plus (iii) zero net torque, allow for a full solution of the problem.

We begin with the torque condition. A convenient choice for the axis of rotation is the bottom end of the rod, since this eliminates one of the unknown forces ($\vec{f}$). With this choice we can readily calculate the torques acting on the rod by using the moment arm expression for the torque, $\tau = r_\perp F$ (Equation 3). We find

$$\sum \tau = T(V) - mg(H) = 0$$

This relation can be solved immediately for the tension, giving

$$T = mg(H/V)$$

Note that the tension is increased if the wire is connected closer to the bottom of the rod; that is, if V is reduced.

Next, we apply the force conditions. First, we sum the y components of all the forces and set the sum equal to zero:

$$\sum F_y = f_y - mg = 0$$

Thus, the vertical component of the force exerted by the bolt simply supports the weight of the lamp:

$$f_y = mg$$

Finally, we sum the x components of the forces and set that sum equal to zero:

$$\sum F_x = f_x - T = 0$$

Clearly, the x component of the force exerted by the bolt is of the same magnitude as the tension, but it points in the opposite direction:

$$f_x = T = mg(H/V)$$

The bolt, then, pushes upward on the rod to support the lamp, and at the same time it pushes to the right to keep the rod from rotating.

For example, suppose the lamp in Figure 6 has a mass of 2.00 kg, and that $V = 12.0$ cm and $H = 15.0$ cm. In this case, we find the following forces:

$$T = mg(H/V) = (2.00\text{ kg})(9.81\text{ m/s}^2)(15.0\text{ cm})/(12.0\text{ cm}) = 24.5\text{ N}$$
$$f_x = T = 24.5\text{ N}$$
$$f_y = mg = (2.00\text{ kg})(9.81\text{ m/s}^2) = 19.6\text{ N}$$

Note that f_x and T are greater than the weight, mg, of the lamp. Just as we found with the diving board in Example 4, the forces required of structural elements can be greater than the weight of the object to be supported—an important consideration when designing a structure like a bridge, an airplane, or a sconce. The same effect occurs in the human body. We find in Problem 25, for example, that the force exerted by the biceps to support a baseball in the hand is several times larger than the baseball's weight. Similar conclusions apply to muscles throughout the body.

In Example 5 we consider another system in which forces have both vertical and horizontal components.

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▲ The chains that support this sign maintain it in a state of translational and rotational equilibrium. The forces in the chains are most easily analyzed by resolving them into vertical and horizontal components and applying the conditions for equilibrium. In particular, the net vertical force, the net horizontal force, and the net torque must all be zero.

REAL-WORLD PHYSICS

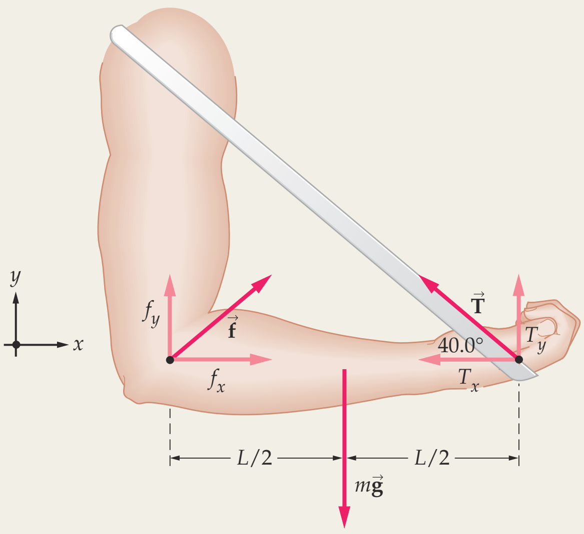
Forces required for structural stability

EXAMPLE 5 ARM IN A SLING

A hiker who has broken his forearm rigs a temporary sling using a cord stretching from his shoulder to his hand. The cord holds the forearm level and makes an angle of 40.0° with the horizontal where it attaches to the hand. Considering the forearm and hand to be uniform, with a total mass of 1.30 kg and a length of 0.300 m, find **(a)** the tension in the cord and **(b)** the horizontal and vertical components of the force, $\vec{\mathbf{f}}$, exerted by the humerus (the bone of the upper arm) on the radius and ulna (the bones of the forearm).

PICTURE THE PROBLEM

In our sketch, we use the typical conventions for the positive x and y directions. In addition, since the forearm and hand are assumed to be a uniform object, we indicate the weight mg as acting at its center. The length of the forearm and hand is $L = 0.300$ m. Finally, two other forces act on the forearm: (i) the tension in the cord, $\vec{\mathbf{T}}$, at an angle of 40.0° above the negative x axis, and (ii) the force $\vec{\mathbf{f}}$ exerted at the elbow joint.



STRATEGY

In this system there are three unknowns: T, f_x, and f_y. These unknowns can be determined using the following three conditions: (i) net torque equals zero; (ii) net x component of force equals zero; and (iii) net y component of force equals zero.

We start with the torque condition, using the elbow joint as the axis of rotation. As we shall see, this choice of axis eliminates f, and gives a direct solution for the tension T. Next, we use T and the two force conditions to determine f_x and f_y.

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SOLUTION

Part (a)

1. Calculate the torque about the elbow joint. Note that f causes zero torque, mg causes a negative torque, and the vertical component of T causes a positive torque. The horizontal component of T produces no torque, since it is on a line with the axis:

$$\sum \tau = (T \sin 40.0°)L - mg(L/2) = 0$$

2. Solve the torque condition for the tension, T:

$$T = \frac{mg}{2 \sin 40.0°} = \frac{(1.30 \text{ kg})(9.81 \text{ m/s}^2)}{2 \sin 40.0°} = 9.92 \text{ N}$$

Part (b)

3. Set the sum of the x components of force equal to zero, and solve for f_x:

$$\sum F_x = f_x - T \cos 40.0° = 0$$
$$f_x = T \cos 40.0° = (9.92 \text{ N}) \cos 40.0° = 7.60 \text{ N}$$

4. Set the sum of the y components of force equal to zero, and solve for f_y:

$$\sum F_y = f_y - mg + T \sin 40.0° = 0$$
$$f_y = mg - T \sin 40.0°$$
$$= (1.30 \text{ kg})(9.81 \text{ m/s}^2) - (9.92 \text{ N}) \sin 40.0° = 6.38 \text{ N}$$

INSIGHT

It is not necessary to determine T_x and T_y separately, since we know the direction of the cord. In particular, it is clear from our sketch that the components of $\vec{\mathbf{T}}$ are $T_x = -T \cos 40.0° = -7.60$ N and $T_y = T \sin 40.0° = 6.38$ N.

Did you notice that $\vec{\mathbf{f}}$ is at an angle of 40.0° with respect to the positive x axis, the same angle that $\vec{\mathbf{T}}$ makes with the negative x axis? The reason for this symmetry, of course, is that mg acts at the center of the forearm. If mg were to act closer to the elbow, for example, $\vec{\mathbf{f}}$ would make a larger angle with the horizontal, as we see in the following Practice Problem.

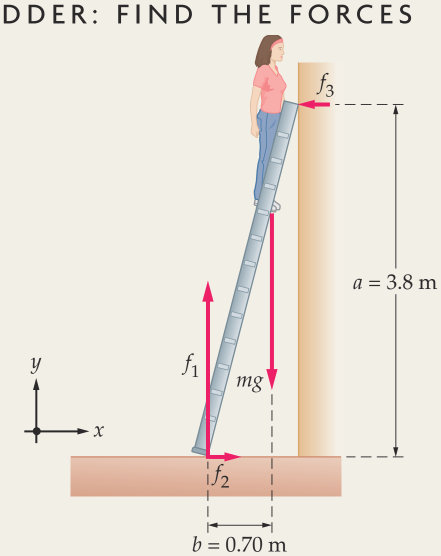
PRACTICE PROBLEM

Suppose the forearm and hand are nonuniform, and that the center of mass is located at a distance of $L/4$ from the elbow joint. What are T, f_x, and f_y in this case? [**Answer:** $T = 4.96$ N, $f_x = 3.80$ N, $f_y = 9.56$ N. In this case, $\vec{\mathbf{f}}$ makes an angle of 68.3° with the horizontal.]

Some related homework problems: Problem 33, Problem 94

ACTIVE EXAMPLE 3 DON'T WALK UNDER THE LADDER: FIND THE FORCES

An 85-kg person stands on a lightweight ladder, as shown. The floor is rough; hence, it exerts both a normal force, f_1, and a frictional force, f_2, on the ladder. The wall, on the other hand, is frictionless; it exerts only a normal force, f_3. Using the dimensions given in the figure, find the magnitudes of f_1, f_2, and f_3.



SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Set the net torque acting on the ladder equal to zero. Use the bottom of the ladder as the axis: $f_3(a) - mg(b) = 0$
2. Solve for f_3: $f_3 = mg(b/a) = 150$ N
3. Sum the x components of force and set equal to zero: $f_2 - f_3 = 0$
4. Solve for f_2: $f_2 = f_3 = 150$ N
5. Sum the y components of force and set equal to zero: $f_1 - mg = 0$
6. Solve for f_1: $f_1 = mg = 830$ N

INSIGHT

If the floor is quite smooth, the ladder might slip—it depends on whether the coefficient of static friction is great enough to provide the needed force $f_2 = 150$ N. In this case, the normal force exerted by the floor is $N = f_1 = 830$ N. Therefore, if the coefficient of static friction is greater than 0.18 [since 0.18(830 N) = 150 N], the ladder will stay put. Ladders often have rubberized pads on the bottom in order to increase the static friction, and hence increase the safety of the ladder.

YOUR TURN

Write both the zero force and zero torque conditions for the case where the axis of rotation is at the top of the ladder.